Stability analysis of a slope subject to real accelerograms by finite elements. Application to San Pedro cliff at the Alhambra in Granada.

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Abstract

The dynamic stability analysis of slopes is often conducted by the traditional method of slices, using pseudo-static calculations. However, the response of a geotechnical structure subjected to seismic loads can be studied through a dynamic finite element analysis, which can be considered one of the most complete available tools, as information about the stress distribution and the deformations can be obtained. The dynamic analysis of the stability of San Pedro cliff at the Alhambra in Granada is studied in this paper. The results have been compared with pseudo-static calculations conducted by the method of slices. Real accelerograms have been selected for the dynamic tests. Thorough in situ and laboratory tests have been conducted in order to properly characterize the cliff. The soil constitutive model is also explained in this paper. Finally, the influence of the sources of energy dissipation has been studied through the material damping, the integration scheme and the boundary conditions.

Keywords. Dynamic calculation, stability of slopes, finite element, accelerogram

1. Introduction

Hanging towns, built at the edge of a cliff and subjected to instability of slopes, are one of the main problems for the conservation of historic sites. An example of this problem is San Pedro cliff, a dihedral 65.5 m high, which has progressed to place itself at a distance of 23.8 m from the Alhambra palace-wall, a World Heritage site (Figure 1. View of San Pedro Cliff cutting the Alhambra hill; at the foot of the hill runs River Darro; to the right the scar of the slab fall in 1985).

Active normal faults surround the cliff and have created an extensional tectonic regime that loosens the ground and actives the fall of slabs. A main fault matches with one of the faces of the dihedral. Stability analyses suggest that the factor of safety of this slope under 1000-yr return period earthquake loading may drop below 1.0 and the critical slip surface could penetrate the Alhambra walls (Justo et al., 2008). A restoration project based on a high-yield-stress wire mesh, post-tensioned by anchors and coloured to blend with the cliff is proposed. The mesh applies a pressure on the slope. This might be the only acceptable solution and its visual impact is moderate.

Most stability analyses still use traditional limit equilibrium approaches involving methods of slices, that have remained essentially unchanged for decades, or pseudo-static calculations, where horizontal forces are used instead of accelerograms. The dynamic calculation by means of the finite element method is accurate, versatile and requires fewer assumptions, especially the failure mechanism (Griffiths and Lane, 1999). Moreover, the dynamic analysis through finite elements can be considered one of the most complete tools to estimate the seismic response of a geotechnical system, as the information about the stress distribution and the deformations/displacements can be obtained. However, it requires an appropriate constitutive model of the soil, a complete description of the soil characteristics and a proper definition of the seismic data. Slope failure occurs naturally through the zones in which the shear strength is insufficient to resist the shear stresses.

The dynamic response of a finite element model is conditioned by the setting of several parameters that influence on the sources of energy dissipation in time-domain analysis. The amount of damping shown by
A discrete numerical system is determined by the material damping, the integration scheme of the equations and the boundary conditions (Visone et al., 2009).

A dynamic analysis of the stability of San Pedro cliff has been carried out with Plaxis 2D, v. 2011. Accelerograms have been selected with the method proposed by Morales-Esteban et al., 2012. The calculation has been repeated for every accelerogram. The information about the geotechnical properties has been obtained from the site investigation conducted by Justo et al., 2008.

2. Background

This section describes the evolution and the causes that have modelled San Pedro cliff. First, a brief description of the geography, the geology and the tectonics of the site is provided, followed by a seismicity analysis of the Granada basin. Later, the origin, evolution and causes of the present situation are discussed, followed by the results of the stability analysis conducted by means of static and pseudo-static calculations. Then, the important environmental and aesthetical issues that surround San Pedro Cliff are described. Finally, a comparative analysis between the proposed solutions and the design solution is shown.

2.1. Geography, geology and tectonic

The Alhambra palace is located on the top of a red hill that dominates the Granada basin (Figure 2. Aerial view of San Pedro Cliff and the Alhambra hill, showing to the north the Albaicín hill and to the south the Cubo tower and the Alhambra Palace; River Darro runs from east to west). San Pedro cliff cuts some dense conglomeratic levels that constitute the Alhambra formation. The Alhambra conglomerates correspond to alluvial fans coming from the erosion of Sierra Nevada, dated upper Pliocene-lower Pleistocene (Justo et al., 2008).

Later (medium and upper Pleistocene), a dense fracturing produced the sinking of the Granada basin. This area was filled with sediments 0.5 million years old.

Faulting mechanisms of current earthquakes show an extensive tectonic regime with main direction NE-SW, compatible with the existence of some active normal faults with strike NW-SE at the surroundings of the Alhambra.

2.2. Seismicity

Morales-Esteban et al., 2012 calculated that the annual rate of earthquakes by square kilometers, in the Granada Basin, is 1.34E-03, the highest in the Iberian Peninsula. There is evidence that important earthquakes occurred in 1431, 1526, 1806, 1911 and 1956 (Azañón et al., 2004).

The Gutenberg-Richter law (Gutenberg and Richter 1942, 1954) relates the cumulative (or absolute) number of events $N(M)$ with magnitude greater or equal to $M$ with the seismic activity, $a$, and the size distribution factor, $b$. A high $b$-value involves that the number of earthquakes of small magnitude is predominant. Contrarily, a low value shows a smaller difference between the relative number of small and large events. The coefficient $b$ usually takes a value around 1.0 and has been widely used by researchers (Utsu, 1999).

$$\log_{10} N(M) = a - bM$$

To solve equation (1) the maximum likelihood method has been applied Aki (1965) and Utsu (1965):

$$b = \frac{\log(e)}{M - M_0}$$

where $\bar{M}$ is the mean magnitude and $M_0$ is the cutoff magnitude (Ranalli, 1969). A value of 1.41 has been obtained for the Granada Basin.

The earthquakes in the Granada Basin are frequent although, normally, of moderate magnitude ($M \leq 6$).

2.3. Origin and evolution

San Pedro cliff has been created by the floods of River Darro, the tectonics, erosion, earthquakes, and, maybe, seepage coming from the palace. The extensive tectonic regime has created fractures that have
favored the erosion generated by the Darro River during floods, modeling a convex curve of the bed through the cliff.

Slides are documented since 1524 when a fire burned the vegetation, leaving the hill unprotected. From Hüfnagel’s engraving of 1564, it can be deduced that the dihedral was located at 60 m from the Alhambra walls and the total height was 33 m. The cited causes of slides are: spills, soil softening produced by water coming from Santa Ana’s aqueduct, explosions, and hillside erosion by seepage coming from the Alhambra (Justo et al., 2008).

2.4. Stability analysis

Justo et al., 2008 have worked out static and pseudo-static calculations of the stability of the slope. The program Geo-Slope and the Morgenstern and Price method, with different pressures applied to the hill, were used. The factor of safety, for the static calculation, is 1.35-1.42 and rises up to 1.55-1.70 for an applied pressure on the mesh of 30 kPa. If no pressure is applied to the slope, the factor of safety under pseudo-static conditions drops below zero, for a pressure of 30 kPa the factor of safety is 1.13-1.17. Figure 3 (Critical slip surface under pseudo-static conditions using the Morgenstern and Price method for a pressure of 30 kPa) shows the critical slip surface calculated with Geo-Slope program. It can be observed that the slip surface could penetrate inside the Alhambra wall placed on the border of the upper part of the cliff.

2.5. Present situation

At present, a series of active faults with throws around 60 cm, cross San Pedro cliff. One of these faults, whose strike is N158º, constitutes the western face of the dihedral. The throw of the eastern face is 7 m.

The extension neotectonics regime produces a reduction of the horizontal stresses in the cliff, which may reach zero value. The joints open in the cliff are now a preferential path for seepage coming from the palace.

Azañón et al., 2004 point out that the Alhambra has remained standing during the past six centuries and that it has a good state of preservation from a seismic point of view. Three reasons are given:

1. Stability resulting from the mechanical behaviour of the conglomeratic formation on which the Alhambra is founded.
2. The moderate magnitude of the seismicity in the Granada Basin.
3. The lack of important seismic active faults in the neighbourhood of the Alhambra.

Justo et al., 2007, have estimated that the average backward displacement of the wedge might be 8 cm/year. Although it might take many years to reach the wall, history shows that the speed of this process is not constant and an important earthquake might accelerate it. Moreover, pseudo-static stability analyses suggest that the critical slip surface could penetrate the Alhambra walls (see figure 3).

2.6 Environmental and aesthetical issues.

San Pedro cliff is placed on the Alhambra hill, where the Alhambra and Generalife rise. In front of it the Albaicín hill stands, where the popular district of the same name is situated (figures 1 and 2). Both hills have been declared World Heritage sites by UNESCO. Darro River runs between them, and the walk by the border of the river, known as Carrera del Darro, is one of the most romantic strolls of the world (Justo et al., 2007). For these reasons, the solution should cause minimum visual impact and should keep the cultural and aesthetic values of the surrounding unaltered.

2.7. Proposed solutions

The danger that implies the progress of the wedge for the Alhambra wall has been foreseen long ago, and from 1520, the following solutions have been proposed and in some cases executed (Justo et al., 2007):

1. Embankments or walls at the foot of the cliff provided to protect the cliff against the floods of the River. Currently, the problem of the floods is not that important, so the construction of embankments is neither necessary nor convenient.
2. Forbid watering the Alhambra forest. This solution would wither the vegetation.
3. To divert the River. This solution, unnecessary today, would have a tremendous environmental impact and would be very expensive.
4. A reinforced earth wall and a double twisted steel wire mesh anchored at the head. This solution would cause a great visual impact on the slope.

5. An ecological wall combined with Californian drains, slope sewing, reinforcement micropiles and acrylic treatment of the slope surface to avoid the erosion (Rodríguez-Ortíz, 1998). This solution would modify considerably the current scenery and the visual impact would be unacceptable.

6. Grouting through a series of steel tubes sub parallel to the slope joined to River regulation. The medium coefficient of permeability of the slope ($2 \times 10^{-7} \text{m/s}$) makes the grouting of the conglomerate difficult. Even more, the grouting pressure might lose slabs from the slope.

### 2.8. Design solution

The solution should be effective and should have minimum environmental impact. The cliff is a unique geological anomaly that should be preserved. To raise the factor of safety up to 1.0 under dynamic conditions and to counteract extensional stress of the cliff, a solution consisting of a high-yield-stress wire mesh, post-tensioned by anchors and coloured to blend with the cliff is proposed. High-yield-stress wire meshes are rhomboidal meshes of galvanized steel lying directly on the slope. The yield stress is 1,770 to 2,020 MPa and the wire thickness 3-4 mm. The pressure on the slope (10-30 kPa) is applied by post-tensioned anchorages isolated or reinforced by 16 to 22 mm thick horizontal cables.

Anchored wire meshes provide a non-expensive method for slope protection and stabilization. For high, steep slopes, they may well be the only possible solution. The mesh and the anchors should raise the factor of safety of the slope. Moreover, the mesh should avoid the erosion of the conglomerate and should provide acceptable visual impact, especially if the present vegetation is maintained.

The structural elements can be colored in ochre and brown to blend with the tonalities of the Cliff. The mesh, unlike conventional galvanized steel meshes, does not reflect the light on the zinc coating. Moreover, the mesh adheres to the original profile of the slope. The mesh thickness is only 3 mm thick and it is very difficult to perceive it. Justo et al., 2007, analyzed the fields of vision from where the restoration is visible and concluded that, once the vegetation has grown, the impact of the project in relation to the aesthetic and cultural values of San Pedro Cliff can be considered moderate.

### 3. Fundamentals

This section exposes all the physical and mathematical fundamentals that support the methodology applied. The choice of the method of slope stability analysis is discussed and the finite element model is presented.

#### 3.1. Choice of the slope stability analysis method

Traditional methods of slope stability analysis use limit equilibrium approaches involving methods of slices that have remained essentially unchanged for decades. The most reputable methods are: Ordinary Method of Slices (Fellenius, 1936), Bishop’s Modified Method (1955), Force Equilibrium Methods (Lowe and Karafiath, 1960), Janbu’s Generalised Procedure of Slices (1968), Morgenstern and Price’s Method (1965) and Spencer’s Method (1967). Griffiths and Lane (1999) point out that these methods are based on the assumption that the failing soil mass can be divided into slices. This hypothesis implies further assumptions relating to side force directions between slices, with consequent implications for equilibrium, which is completely an artificial concept.

Pseudo-static calculations substitute real accelerograms, whose value and direction of acceleration change instantly, for a static horizontal force. Also the dynamic properties of a soil, for example Young’s modulus, differ from static to dynamic conditions.

Jing (2003) presents a very detailed work on techniques, advances and outstanding issues in numerical modelling for rocks. Three main groups can be distinguished: a) continuum methods, a1) finite difference method (FDM), a2) finite element method (FEM) and a3) boundary element method; b) discontinuum methods, b1) discrete element method (DEM) and discrete fracture network method (DFN); c) hybrid continuum/discontinuum models, c1) hybrid FEM/BEM, hybrid DEM/BEM and c3) other hybrid models. Jing (2003) points out that there are no absolute advantages of one method over another.

The continuum criteria referred here is a macroscopic concept. Continuity implies that at all the points the materials cannot be torn open or broken into pieces (Jing 2003). All material points should remain in the
same neighbourhood throughout the deformation. A microscopic scale is neither necessary in practice nor mathematically advisable.

The FEM is perhaps the most widely applied numerical method in rock mechanics and rock engineering today. Compared with traditional methods, it presents some advantages: the failure surface occurs through the zones where the applied shear stresses exceed the soil shear strength, the failure occurs naturally and no assumptions about its shape and location are previously made; global equilibrium is kept until failure is reached, no slices are supposed and no hypothesis about slide side forces is needed; the finite element method provides information about deformations at working stress levels; and progressive failure can be monitored up to and including overall shear failure. The FEM is accurate, versatile and flexible in handling material heterogeneity, non-linearity and boundary conditions. Finally, in a dynamic analysis the information about the stress distribution and the deformation/displacements versus time can be obtained.

The in situ tests (Justo et al., 2008), that will be described later in section 4.1, show that the layers of the Alhambra conglomerate can be modelled as a continuum material. For all the aforementioned, the continuum FEM has been used in this study.

3.2 Finite element model

A Mohr-Coulomb ground model under drained conditions has been chosen.

The basic element used in this analysis is the triangular element of 15 nodes. It provides a fourth order interpolation for displacement and the numerical integration involves 12 Gauss points (stress points). It is a very accurate element that produces high quality stress results. The mesh density is fine.

The average element size (AES) represents the average side length of the element used and is an average dimension, which is representative of the mesh thickness.

Every time a numerical analysis is calculated, the influence of the mesh must be checked. Kuhlmeyer and Lysmer (1973) suggested that the element size should be no larger than $\lambda/8$, where $\lambda$ is the wave length with the maximum frequency of interest, $f_r$. In this study, $\lambda/8 = V_s/8f_r = 12.5$ m, where $V_s = 800$ m/s and $f_r = 8$ Hz. In this analysis the AES is 6.56 m.

Tables 1 and 2 quantify the properties of the layers that form the Alhambra conglomerate used in the dynamic analysis. Figure 4 (Layers of San Pedro Cliff with the mesh and anchors proposed; the effect of the Cubo tower, located at the top of the slope, is simulated as a continuum load) shows the layers of the cliff with the anchors and the proposed mesh. Figure 5 (Discretization in finite elements and points used for the dynamic analysis) plots the points used in the analysis and the finite element discretization, automatically done by the program, for a fine global coarseness.

4. Calculation parameters

The commercial program Plaxis (Plaxis 2D, v. 2011) has been used for the dynamic finite element calculations. As mentioned before, three requirements must be fulfilled in order to conduct a realistic dynamic analysis: First, a thorough soil characterization of the geotechnical structure must be performed; second, the soil constitutive model is described; third, the seismic input data selected are presented. Moreover, the sources of energy dissipation are also discussed. Finally, the calculation hypotheses are listed.

4.1. Soil characterization

Ten boreholes were drilled in the upper, medium and lower parts of the cliff, to a maximum depth of 45 m. In two of the boreholes drilled at the top of the escarpment, pressuremeter, down-hole, cross-hole, penetration, permeability and laboratory tests were carried out (Justo et al., 2008). Tables 1 and 2 present the properties of the layers that appear from top to bottom in the geological profile:

1. Dense conglomerate, with 100 mm maximum particle size and core recovery 100%, of brown to pale gray silty matrix.
2. Very dense conglomerate, with maximum particle size of 5-8 cm and core recovery 100%, of brown to reddish silty to clayey matrix.
3. Moderately dense conglomerate, with core recovery 60%, of brown to pale gray silty matrix.
4. Very dense, gravelly and sandy conglomerate, of brown to pale gray silty fine matrix and variable permeability.
4a. One meter thick clay layers, interspersed in layer 4. Core recovery 100%.

Talus appears at the foot of the slope. It is composed of quartzose and phyllitic blocks, gravel and sand, with a predominance of the sand fraction.

The well-documented slab failure of 1985 indicates that slides usually occur on one of the dihedral sides and are not three-dimensional. Although there is clast orientation in the direction of the fault, the granular nature of the conglomerate guarantees that there is no great decrease in this direction, as would happen in clay, shale or slate.

Boore et al., 1993 classifies the local geology according to the average velocity of the shear waves measured or estimated at a depth of 30 m. In the Alhambra conglomerate the lowest transverse wave velocity measured is 800 m/s for layers 1, 3 and 4a (table 1) and corresponds to rock ($V_s \geq 750$ m/s).

4.2. Soil constitutive model

Although a great number of failure criteria to model the strength of the ground have been proposed, the Mohr-Coulomb’s criterion remains the most widely used in geotechnical practice (Griffiths and Lane, 1999). A non-associated, perfect-plasticity Mohr-Coulomb model has been selected in this paper. The model is widely known and needs no description.

If the stresses lie on or outside the failure envelope (see Figure 6. Mohr-Coulomb yield surface in principal stress space), then that Gauss-point is assumed to be yielding. Yielding stresses are redistributed throughout the mesh using the visco-plastic algorithm (Perzyna 1966, Zienkiewicz and Corneau, 1975). Overall shear failure occurs when a sufficient number of Gauss-points have yielded to allow a mechanism to develop. Hence, besides the plasticity parameters $c$, $\varphi$ and $\psi$, the elasticity parameters Young’s modulus, $E$, and Poisson’s ratio, $\nu$, are required (see tables 1 and 2).

4.3. Seismic input

The method described in (Morales-Esteban et al., 2012) has been used in order to select accelerograms. This method is based on constructing a uniform seismic hazard acceleration response spectrum (USHARS), according to the type of soil and the required hazard level (exposure time and exceedance probability) at the site the structure is located. Then, the standard deviation between the response spectrum of real accelerograms and the calculated response spectrum is calculated by means of equation (12) in Morales-Esteban et al., 2012. The five accelerograms whose standard deviation is lower, and in which SMC format is available for both axis, have been selected. Table 3 summarizes the accelerograms used in the dynamic calculations and its standard deviation ($\sigma$). The selected accelerograms were recorded in stations placed on rock ($V_s \geq 750$ m/s). Their response spectrum is equivalent to a 5 % probability of exceedance and a time of exposure of 50 years, which is equivalent to a return period of 974 years. The applied ground motion corresponding to the Lazio Abruzzo aftershock (record 990, 07/05/1984) recorded at the Atina-Pretura Terrazza station in Italy is plot in Figure 7 (Lazio Abruzzo aftershock, record 990, x-axis acceleration time-history).

4.4. Sources of energy dissipation

Visone et al., 2009 perform an analysis of the calibration of two-dimensional finite elements models in geotechnical earthquake engineering through the comparison between the one-dimensional vertical propagation of S-waves in elastic layers, whose theoretical solutions are available in literature, with a finite element analysis. They conclude that in time domain seismic analyses the response of FEM is conditioned by the sources of energy dissipation and give some preliminary advances on how to calibrate these parameters.

4.4.1. Material damping

In time domain analysis material damping includes viscous and hysteretic soil damping and is frequency dependent. The material damping is simulated by means of the well-known Rayleigh formulation (Rayleigh and Lindsay, 1945). The damping matrix, $C$, is assumed to be proportional to the mass matrix, $M$, and to the stiffness matrix, $K$, through the coefficients $\alpha_R$ and $\beta_R$ according to the following relation:

$$ C = \alpha_R M + \beta_R K $$

(3)
With respect to frequency, the dynamic response of a system is strongly related to the value of these parameters. It should be noted that this formulation is not the best to simulate damping but it is commonly used in numerical codes as it is straightforward to implement. Some studies have been performed in order to establish the damping of a layer of soil and to determine Rayleigh’s coefficients (Park and Hashash, 2004; Visone et al., 2009).

Park and Hashash, 2004 deeply studied the formulation of the soil damping. For three columns of 100 m, 500 m and 1000 m, representative of the Mississippi Bay, they used two damping ratios, one constant (1.8%) and another decreasing with depth. Visone et al., 2009 used a 2% constant damping ratio for a visco-elastic homogenous layer that lies on rigid bedrock; following this author, a 2% constant damping, $D$, has been used in this study.

4.4.2. Numerical damping (integration scheme)

The formulation of the integration with time constitutes an important factor in the stability and accuracy of the calculation process. Implicit and explicit methods are used. The advantage of the explicit integration is that is relatively simple to formulate. However, the disadvantages are that the calculation process is not robust and several limitations are imposed in the calculation steps. Implicit methods are more complicated but produce a more reliable calculation process and normally more accurate. In this study, the Newmark’s implicit time integration system is used. With this method, the displacement and the velocity at the point in time $t + \Delta t$ are expressed, respectively as:

$$u^{t+\Delta t} = u^t + \dot{u}^t \Delta t + \left(\frac{1}{2} - \alpha_N\right) \ddot{u}^t + \alpha_N \ddot{u}^{t+\Delta t} \Delta t^2$$ (4)

$$\dot{u}^{t+\Delta t} = \dot{u}^t + \left(1 - \beta_N\right) \ddot{u}^t + \beta_N \ddot{u}^{t+\Delta t} \Delta t$$ (5)

where $\Delta t$ is the time step.

The coefficients $\alpha_N$ and $\beta_N$ control the accuracy of the numerical time integration. These coefficients should not be confused with the Rayleigh coefficients. In order to obtain a stable solution, the following condition must be considered:

$$\beta_N \geq 0.5, \alpha_N \geq \frac{1}{4} \left(1 + \beta_N\right)^2$$ (6)

Hilbert et al., (1977) proposed a change in the Newmark scheme, named the Newmark HHT modification. The new Newmark parameters are now expressed as a function of the parameter $\gamma$ that is a numerical dissipation parameter:

$$\alpha_N = \frac{(1+\gamma)^2}{4}, \beta_N = \frac{1}{2} + \gamma$$ (7)

where $\gamma$ varies between 0 and 1/3.

If $\gamma = 0$, the coefficients coincide with those of the original Newmark method with constant average acceleration. When $\gamma > 0$, the efficiency of the calculation is improved, but a numerical source of damping is introduced into the model.

Despite the advantages of the implicit integration, it is subjected to some limitations due to the time step used in the calculation. If the time step is too large the solution will display substantial deviation and the calculated response will be unreliable. The calculations have been done considering a time step, $\Delta t$, equal to 0.05 ms in order to respect the following rule on the critical time step for a single mesh element (Brinkgreve, 2002):

$$\Delta t_{\text{crit}} = \frac{l_e}{a} \sqrt{\frac{c_1(1-\nu)}{\nu(1+\nu)(1-2\nu)\left[\frac{1}{12} \overline{E} + \frac{1}{12} \overline{\nu}^2 \overline{E} \left[1 + 1.5\overline{\nu} \right] + \frac{1}{4} \overline{\nu}^2 \right]}}$$ (8)

The critical time step depends on the maximum frequency and the coarseness of the finite element mesh. The parameter $l_e$ denotes the average length of an element. The factor $a$ depends on the element type. For a 15-node element $n=1/19(c15)^{0.5}$, with $c15=4.9479 \times 10^{-3}$ (Zienkiewicz and Taylor, 1991). $B$ and $S$ are, respectively, the largest dimension and the surface area of the finite element.
Plaxis allows the input of accelerograms by means of a file with ASCII or SMC format. For that purpose a displacement as a function of time is applied to the basement, obtained from the integration of the accelerograms through Newmark’s integration method. The horizontal acceleration of a point, situated on the top of the cliff and subject to the application, in the basement, of accelerogram 990, can be observed in Figure 8 (Horizontal acceleration versus time for a point placed at the top of the cliff, subject to accelerogram 990). Accelerogram 990 has been previously transformed into displacements by means of the Newmark’s integration scheme.

This method has the difficulty of setting the appropriate coefficients for the actual damping of the numerical model. Visone et al., 2009 suggest that a possible solution to limit this uncertainty is to set the minimum value for Newmark, γ, which allows stability, then fit the theoretical solution. To avoid introducing numerical dissipation in the numerical damping the following value has been used: γ = 0 (α_N = 0.25 and β_N = 0.5).

In order to obtain the Rayleigh coefficients, the method proposed by Visone et al., 2009 has been followed. They compare the vertical one-dimensional propagation of shear waves in visco-elastic homogenous layer that lies on rigid bedrock, whose theoretical solution is known with the results obtained from the finite element method. They conclude that if no numerical dissipation is introduced in the time integration scheme, the damping of the system can be modelled by using the Rayleigh coefficients only. The following formulation in provided by Visone et al., 2009, for a soil layer with a constant damping ratio, D, to obtain α_R and β_R:

\[
\alpha_R + \beta_R \omega_n^2 = 2\omega_n D
\]  

(9)

where \( \omega_n \) are the circular natural frequencies of the layer:

\[
\omega_n = 2\pi f_n
\]

(10)

And the natural frequencies of the system can be obtained as:

\[
f_n = \frac{\nu_i}{4k_i}(2n - 1)
\]

(11)

In which \( n \) is order number of the calculated natural frequency.

However, in this case study an additional difficulty appears as there are four layers superimposed on rigid bedrock. To solve this limitation the EERA (2000) code has been used and a unique equivalent layer over rigid bedrock has been used to obtain the Rayleigh coefficients. An average shear wave velocity, \( V_{sm} \), of the soil profile is defined as:

\[
V_{sm} = \frac{1}{\sum_i h_i} \sum_i h_i V_i
\]

(12)

The fundamental period, \( T \), of the equivalent soil profile is calculated as:

\[
T = 4 \sum_i h_i V_i^{-1}
\]

(13)

In table 4, the two first natural frequencies of the equivalent layer are listed. From the amplification function, plot in Figure 9 (Amplification function of the equivalent visco-elastic layer over bedrock) the natural frequencies of the seven first natural frequencies can be obtained. The Rayleigh coefficients shown in Table 4 have been used in all layers.

**4.4.3. Boundary conditions**

In order to avoid the effects due to the reflection of waves on the boundaries, absorbent boundaries have been defined. A damper is used instead of applying fixities in a certain direction. The damper ensures that an increase in stress on the boundary is absorbed without rebounding.

The use of absorbent boundaries is based on the method described by Lysmer and Kuhlmeyer (1969). The normal and shear stress components absorbed by a damper in x-direction are:

\[
\sigma_n = -C_t \rho V_p \dot{u}_x
\]

(14)
\[ \tau = -C_2 \rho V_p \dot{u}_p \]  

(15)

Where, \( \rho \) is the density, \( V_p \) and \( V_s \) are the pressure wave and the shear wave velocities, respectively. \( C_1 \) and \( C_2 \) are relaxation coefficients that have been inserted in order to improve the effect of the absorption. To the authors’ knowledge there is not a well-established criterion in the literature to determine the value of the relaxation coefficients. Brinkgreve (2002) suggested the use of \( C_1 = 1 \) and \( C_2 = 0.25 \). However, it is not possible to state that shear waves are fully absorbed so that in presence of shear waves a limited boundary effect is noticeable. To solve this issue, lateral boundaries are placed sufficiently far away from the central zone. In this study, the discretized zone has been enlarged to thrice the width used in static or pseudo static calculations (compare Figure 4 with Figure 3).

4.5 Calculation hypotheses

The following calculation hypotheses have been considered:

1. Own weight of the cliff.
2. Own weight + mesh and anchors.
3. Own weight + mesh and anchors + post-tension (125 kN).
4. Own weight + mesh and anchors + post-tension (250 kN).
5. Own weight + mesh and anchors + post-tension (375 kN).
6. Own weight + mesh and anchors + post-tension (448 kN).
7. Own weight + seismic acceleration given by Spanish standard (SAS).
8. Own weight + mesh and anchors + SAS.
9. Own weight + mesh and anchors + SAS + post-tension (125 kN).
10. Own weight + mesh and anchors + SAS + post-tension (250 kN).
11. Own weight + mesh and anchors + SAS + post-tension (375 kN).
12. Own weight + mesh and anchors + SAS + post-tension (448 kN).
13. Own weight + USHARS accelerogram.
14. Own weight + mesh and anchors + USHARS accelerogram.
15. Own weight + mesh and anchors + USHARS accelerogram + post-tension (125 kN).
16. Own weight + mesh and anchors + USHARS accelerogram + post-tension (250 kN).
17. Own weight + mesh and anchors + USHARS accelerogram + post-tension (375 kN).
18. Own weight + mesh and anchors + USHARS accelerogram + post-tension (448 kN).

The anchors are separated 5m. In the plane strain calculations, the four anchor loads correspond to 25, 50, 75 and 89.6 kN/m.

5. Experimental results

The most relevant results obtained from all the hypotheses calculated are shown in this section. First, the pseudo-static results are exposed. Second, the stress is analyzed by means of the relative shear stresses. Third, the plastic points and tension cut-off are shown. Finally, an analysis of the deformation/maximum displacements is conducted.

5.1. Pseudo-static results

The factor of safety of San Pedro cliff at the Alhambra in Granada obtained with a static calculation, using PLAXIS is 1.34. With the high-yield-stress wire mesh and the post-tensioned anchors the factor of safety raises up to 1.55. For the pseudo-static calculations, the ground acceleration applied to the slope corresponding to the 1,000-year return period (0.28g), following the recommendations of the Spanish standard for monuments, is 0.28g. With the mesh and the anchors the factor of safety is 1.01. Four different anchor forces have been applied (125, 250, 375 and 448 kN/m). The calculations show that the improvement produced by an anchor force larger than 125 kN/m is negligible: the plastic points, the relative shear stresses and the deformations are not significantly reduced. However, due to tectonic considerations, a pre-stress of 448 kN has been finally adopted.

5.2 Dynamic results

5.2.1. Analysis of stress

Relative shear stresses indicate the proximity of stress to the failure envelope. The relative shear stress, \( \tau_{rel} \), is defined as:
\[ \tau_{rel} = \frac{\tau}{\tau_{max}} \]  

(16)

where \( \tau \) is the maximum value of shear stress and \( \tau_{max} \) is the shear strength.

Unless otherwise stated, all the graphs shown below correspond to the following conditions:

1. When post-tension is employed, the anchor load is 89.6 kN/m.
2. The true dynamic calculations are for accelerogram 990, and correspond to the instant with higher stresses or deformations.

The comparison between Figure 10 (Relative shear stresses of San Pedro cliff, in a static calculation; no anchors, no mesh) and Figure 11 (Relative shear stresses of San Pedro cliff, with mesh and post-tensioned anchors in a static calculation) shows that the area where the relative shear stresses exceed 0.90 is fully covered by the anchors. This shows the good disposal of the anchors. It can also be observed that no slip surface is formed and that if fall of slabs would happen it would only be superficial. Figure 12 (Relative shear stresses of San Pedro cliff, with mesh and post-tensioned anchors; dynamic calculation) plots the instant, where the relative shear stresses are higher for the 5 accelerograms calculated. The FE method allows analyzing at every instant a geotechnical system subject to seismic loads. In this case, all the images have been analyzed and the most relevant one is shown. Although a slip surface is hinted, it does not reach the top of the cliff and the anchors saw the area with higher relative shear stresses. In Figures 10 to 12, the higher relative shear stresses are concentrated in the most superficial area of the slope.

The FE calculations also allow checking the anchors’ stress variation versus time in order to verify that none of them exceeds the allowable stress (100 kN/m).

Figure 13 (Variation of the anchors’ stress during the dynamic calculation) shows the variation of the anchors’ load with time. It can be observed that the original post-tension is 89.6 kN/m and that the anchors do not exceed the allowable stress (100 kN/m). It should be noted that the variation of the anchors’ load is less than 3% of its initial load.

5.2.2. Analysis of the Mohr-Coulomb points and tension cut-off points

Mohr-Coulomb points are the stress points that lie on the surface of the Mohr-Coulomb failure envelope. Tension cut-off points are the points where the tension cut-off criterion has been reached (in this case for zero tension). The comparison between Figure 14 (Mohr-Coulomb points and tension cut-off points at San Pedro cliff, in a static calculation; no anchors, no mesh) and Figure 15 (Mohr-Coulomb points and tension cut-off points in San Pedro cliff, with mesh and post-tensioned anchor load, in a static calculation) shows that the pressure applied to the slope reduces the Mohr-Coulomb points for the static conditions. In Figure 16 (Mohr-Coulomb points and tension cut-off points of San Pedro cliff, with mesh and post-tensioned anchor load, in a dynamic calculation), the Mohr-Coulomb points are represented for the image of the five calculated accelerograms where the number of them is higher. Similarly to Figure 12, it can be observed that an incipient slip surface is formed but not completed, and only superficial fall of slabs might be expected.

5.2.3. Analysis of the deformation

The finite element method allows the user to observe the deformation versus time of a geotechnical structure subjected to accelerograms. Figure 17 (Deformed mesh of San Pedro cliff, with post-tensioned anchors and mesh, in a dynamic calculation) shows the deformed mesh of the cliff in the instant corresponding to Figures 12 and 16, that is, when the relative shear stresses are higher and when the number of Mohr-Coulomb points is larger. It can be observed that the deformations in the vertical direction are moderate. Figure 18 (Time history of horizontal displacements at points A and B in dynamic calculation) plots the horizontal displacements of points A and B (Figure 5), this last placed at the base, under the application of accelerogram 990 to the base of the structure for 15s. It is important to study the displacements of the points in relation to a point in the base in order to obtain the deformation as the difference between the studied point and the point in the base.

6. Conclusions

San Pedro cliff at the Alhambra in Granada is relevant due to the aesthetical and cultural values that surround it. A historical analysis has shown that, in the 16th century, it was situated at 60 m from the Alhambra walls and that its total height was 33 m. However, nowadays it has approached up to 23.8 m and has grown up to 65.5 m height. A solution is necessary to preserve the Alhambra hill, the Alhambra
walls and the Cubo Tower, placed on the top of it. A high-yield-stress wire mesh, post-tensioned by anchors and coloured is proposed. It may be one of the fewest possible solutions to protect the slope and to raise the factor of safety with a minimum environmental impact.

The analysis of the stability of the slope using the Morgenstern and Price method has shown that the factor of safety for static conditions is 1.35-1.42, which rises up to 1.55-1.70 if a pressure of 30 kPa is applied to the slope. The pseudo-static calculations have demonstrated that the cliff could be unstable if the seismic acceleration given by Spanish standard (SAS) is applied. However, a pressure of 30 kPa would raise the factor of safety up to 1.13-1.17. These calculations have shown that the slip surface could penetrate inside the Alhambra wall.

An analysis of the stability of the cliff has been conducted with the finite element method. The static calculations suggest that the factor of safety is 1.34. With the mesh and the post-tensioned anchors it raises up to 1.55, similarly to the results obtained with the Morgenstern and Price method. The pseudo-static calculations under the SAS have demonstrated that the factor of safety would only be 1.01 with the mesh and the post-tensioned anchors.

Moreover, the response of the cliff under seismic loading has also been studied by means of a dynamic finite element calculation. The calculations have been repeated with five different real accelerograms recorded over rock. The response spectrum of the five accelerograms selected is equivalent to a return period of 974 years. The criteria used to calibrate the dynamic parameters are shown in the text.

The analysis of the results (with mesh and post-tensioned anchors) shows that an incipient slip surface is formed but it does not reach the top of the cliff. This suggests, in concordance with the pseudo-static calculation obtained by the Morgenstern-Price and finite element method, that the reinforcements of the slope is essential to guarantee its stability. The dynamic calculations have shown that the fall of slabs would only be superficial, similarly to the slab failure of 1985. This result is slightly different to the critical slip surface obtained with the Morgenstern-Price method that could penetrate inside the Alhambra walls.

For all these reasons the solution of the mesh and the post-tensioned anchors is essential to preserve the cliff. The mesh would protect the surface of the slope from erosion and fall of slabs. The pressure applied would counteract the extensive tectonic regime and reduce the formation of plastic points. It has also been demonstrated that without reinforcement the slope could be unstable under seismic loads.

Acknowledgements

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http://dx.doi.org/10.1080/13632460902988950


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Table 1. Average properties for the layers of the Alhambra conglomerate. f.d. = free drainage; est. = estimated value; \( p_i^* \) = net pressuremeter limit pressure; \( E_M \) = deformation modulus; \( E_p \) = pressuremeter modulus; \( V_P \) = longitudinal wave velocity; \( V_s \) = transverse wave velocity; \( E_d \) = dynamic modulus; \( \rho_0 \) = uniaxial compressive strength.
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Table 2. Calculation values. 1) moderately dense conglomerate; 2) conglomerate with gravel; 3) dense conglomerate; 4) very dense conglomerate.
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Table 3. Information about the seismic data of records whose standard deviation (σ) is minor in relation to the uniform seismic hazard acceleration response spectra for San Pedro cliff.
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Table 4. Raileigh coefficients and the two first natural frequencies for the equivalent San Pedro Cliff layer considering a 2% constant damping ratio.
Figure 1
Click here to download high resolution image
Relative shear stresses
Figure 12
Relative shear stresses
Deformed mesh $|u|$ (increased 50 times)