Risk-based optimal bidding strategy of generation company in day-ahead electricity market using information gap decision theory

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ABSTRACT

This paper considers a price-taker generation company to participate in day-ahead electricity energy market. While making optimal bidding strategy for producer, factors such as the characteristics of generator and the market price uncertainty need to be considered because of having direct impact on the expected profit and bidding curve. The market price considered an uncertain variable and it is assumed that the generation company forecasted the market prices. In this study, the uncertainty model of market price is considered based on the concept of weighted average squared error using a variance–covariance matrix. Information gap decision theory is used to develop the bidding strategy of a generation company. It assesses the robustness/opportunity of optimal bidding strategy in the face of the market price uncertainty while producer considers whether a decision risk-averse or risk-taking. It is shown that risk-averse or risk-taking decisions might affect the expected profit and bidding curve to day-ahead electricity market. A case study is used to illustrate the proposed approach.

1. Introduction

In a day-ahead energy auction, generation companies submit supply bids and energy service companies submit hourly demand offers for the next day to market operator. Supply bids and demand offers can be submitted in the form of a set of price-quantity ($/MW h MW) pairs. The market operator processes the supply bids and the demand offers and computes the price that clears the market as well as the trading volume.

The complexity of the problem that a generation company faces in preparing its optimal bidding strategy arises from the fact that it must take a decision based on imperfect information on the market prices. In imperfect electricity market, a generation company could develop bidding strategies to maximize its own expected profits. A generation company has to make a decision based on limited information. For example, a generation company does not know the actual system market clearing price (MCP) beforehand since it depends on bidding behavior of other participants in the market. Thus, an optimal bidding strategy is a challenging task for generation companies. This uncertainty implies that the market price is unknown for generation companies. Therefore, generation companies should forecast the market prices.

Information gap modeling is a stark theory of uncertainty, motivated by severe lack of information. It does, however, have its own particular subtlety. It is facile enough to express the idea that uncertainty may be either pernicious or propitious. That is, uncertain variations may be either adverse or favorable. Adversity is the possibility of failure while favorability is the possibility of sweeping success [1].

In this paper, the optimal bidding strategy problem affected by the uncertainty of market price is formulated and solved using the information gap decision theory (IGDT). IGDT modeling is put forth as a basic approach for enhancing decision making under uncertainty, especially when there is a high level of uncertainty and little information is available.

1.1. Literature review of optimal bidding strategy

The optimal bidding strategy problem optimization models include many mathematical programming methods. As reviewed in [2], a method for simultaneous bidding is proposed, where the bidding prices and quantities on the spot and reserve markets are calculated by maximizing a stochastic nonlinear objective function of expected profit. A new method to obtain the equilibrium points for reliability and price bidding strategy of units is proposed in [3] when the unit reliability is considered in the scheduling problem. In [4], the linear supply function model is applied so as to find the supply function equilibrium analytically. It also proposed a new and efficient approach to find SFEs for the network constrained electricity markets by finding the best slope of the supply function with the help of changing the intercept. A new method that uses the combination of particle swarm optimization
(PSO) and simulated annealing (SA) to predict the bidding strategy of generating companies in an electricity market is proposed in [5]. In [6], the optimal bidding strategy of wind power producer in adjustment spot markets is addresses, in order to increase the wind producer revenues through a stochastic optimization process which considers the uncertainty of the random variables involved, namely short-term wind power prediction, intra-day market price prediction and imbalance price prediction. The self-scheduling problem of the power company is a profit maximization problem, which is formulated and solved as a mixed integer linear program (MILP) in [7], considering the co-optimization of energy as well as primary (up/down), secondary (up/down) and tertiary (spinning/non-spinning) reserve markets in a pool-based framework similar to the Greek day-ahead market structure. In [8], a probabilistic Price Based Unit Commitment (PBUC) approach using Point Estimate Method (PEM) is employed to model the uncertainty of market price and generation sources for optimal bidding of a virtual power plant (VPP) in a day-ahead electricity market. In [9], a stochastic mixed-integer linear programming model for determining the optimal bidding strategy considering the uncertain market prices is proposed. A methodology for profit maximizing bidding under price uncertainty in a day-ahead is proposed in [10] and multi-unit and pay-as-bid procurement auction for power systems reserve is proposed and market price uncertainty is modeled using the probability distribution. The bidding decision making problem in electricity pay-as-bid auction is studied in [11] from a supplier’s point of view. The market clearing price (MCP) is considered as a continuous random variable with a known probability distribution function (PDF), and an analytic solution is proposed. In [12], a stochastic multi-objective model for self-scheduling of a power producer is presented and the price forecasting inaccuracies are modeled as scenarios tree using a combined fuzzy c-mean/Monte-Carlo Simulation (FCM/MCS) method. An evolutionary imperfect information game approach to analyze the bidding strategy in electricity markets with price-elastic demand is proposed in [13]. A new method for determination of the optimal bidding strategies among generating companies in the electricity markets using agent-based approach and numerical sensitivity analysis is presented in [14]. An optimal risky bidding strategy for a generating company by self-organizing hierarchical particle swarm optimization with time-varying acceleration coefficients (SPSO–TVACs) is discussed in [15]. In [16], a new genetic algorithm is proposed for determining the optimal bidding strategy of generating company in the day-ahead electricity market. This approach is developed from viewpoint of a generating company that participates in a market and wish to maximize its own profit. In [17], a mixed-integer nonlinear programming approach is considered to enable optimal hydro scheduling for the short-term time horizon, including the effect of head on power production, start-up costs related to the units, multiple regions of operation, and constraints on discharge variation. In [18], a new stochastic framework for clearing of day-ahead reactive power market is presented. The uncertainty of generating units in the form of system contingencies are considered in the reactive power market-clearing procedure by the stochastic model in two steps. The Monte-Carlo Simulation (MCS) is first used to generate random scenarios. Then, in the second step, the stochastic market-clearing procedure is implemented as a series of deterministic optimization problems (scenarios) including non-contingent scenario and different post-contingency states. In [19], the coordinated interaction between units’ output and electricity market prices, the benefit/risk/emission comprehensive generation optimization model with objectives of maximal profit and minimal bidding risk and emissions is established.

1.2. Literature review of market price modeling

In the technical literature, several techniques to forecast short-term electricity prices have been reported, namely hard and soft computing techniques.

The hard computing techniques include autoregressive integrated moving average (ARIMA) [20], wavelet-ARIMA [21], time series model [22], and mixed model [23] approaches. The soft computing techniques include neural networks (NNs) [24], fuzzy neural networks (FNNs) [25], weighted nearest neighbors (WNNs) [26], adaptive wavelet neural network (AWNNe) [27], hybrid intelligent system (HIS) [28], neuro-fuzzy models [29], and cascaded neuro-evolutionary algorithm (CENA) [30]. A combination of neural networks with wavelet transform (NNWT) has also been recently proposed [31], presenting a good trade-off between forecasting accuracy and computation time.
IGDT, which makes minor assumptions on the structure of uncertainty, is developed in [1] as an alternative approach for decision making under uncertainty. The IGDT models do not use the measure functions such as probabilistic density, or fuzzy membership functions. In other words, it models the error between the actual and the forecasted variable. In this paper, IGDT is used as a tool to model the uncertainty of market price of optimal bidding strategy for a generating company.

In the current work, IGDT approach addressed the question of how much uncertainty of market price can affect optimal bidding strategy of producers. This approach assesses the robustness/opportunity of optimal bidding strategy in the face of uncertainty of market price while the producer is considering whether a decision risk-averse or risk-taking. The issue which is addressed here is to determine an optimal bidding strategy for a producer considering one day planning horizon. The risk associated with this issue is modeled based on the concepts derived from IGDT approach. For this purpose, a risk-averse model is proposed to find robust strategies through maximizing the robustness function of the optimal bidding strategy against the immunity to windfall casted market prices or profits lower than the expected profit. Furthermore, a risk-taking model is proposed to obtain the benefits of high market prices through minimizing the opportunity function of optimal bidding strategy against the immunity to windfall due to high forecasted market prices or profits higher than the expected profit. It should be noted that IGDT is used to find the optimal procurement strategy of large consumers in [32–35].

The remainder of the paper is organized as follows. The next section describes the expected profit of a generation facility and the problem formulation of the risk-based optimal bidding strategy using IGDT approach. As mentioned, the solution methodology for price-taker thermal unit is highlighted in this paper. For test results, a numerical example is used to illustrate the bidding approach. Afterwards, the paper is concluded. Finally, an Appendix provides a background on IGDT.

2. Problem formulation

In this section, the profit function formulation of a generation company subject to its related constraints is modeled and the robustness and opportunity functions based on IGDT are derived.

2.1. Assumptions

In this paper, the following assumptions are considered.

(1) The generating company has a thermal unit and participates in day-ahead energy market.
(2) The unit participates with its entire available capacity in day-ahead energy market.
(3) The unit acts as a price-taker, believing that its energy bid cannot influence the market price.
(4) The unit has a forecasted market price.
(5) The unit has a deterministic estimate of variance–covariance matrix between hourly market prices.
(6) The unit has perfect information about its technical constraints.
(7) The price–energy curve is a non-decreasing function.
(8) Minimum up time and minimum down time constraints and start-up and shutdown costs are considered.

2.2. Market price modeling

In this paper, the generation company acts as a price-taker, believing that his energy bid cannot influence the market price. The market price is determined based on the intersection of bidding curves with the required demand. Also it is considered that the payment will be based on market clearing price (MCP). So, the generation company must forecast and use it as input variable to his optimal bidding strategy problem. In this case, the market uncertainty is fully described by the market price uncertainty, which affects the bidding strategy of the generation company. Some backgrounds about price forecasting methods are addressed in Section 1.2.

The market prices are indicated by

\[ \lambda = [\lambda_1, \ldots, \lambda_T]. \]  

Since hourly prices are random variables, they can be described as

\[ \lambda_t = \bar{\lambda} + e_t \quad \forall t = 1, \ldots, T \]

where \( e_t, \forall t \), depicts the error or difference between the actual and forecasted prices at time \( t \). These errors can be organized as

\[ e = [e_1, \ldots, e_T]. \]  

2.3. Decision variable

The decision variables in bidding strategy contain the price-quantity pairs. The price will be defined based on (2) after optimizing the objective functions. The quantity is the amount of power which should be produced from thermal unit at different times. This variable can be defined as:

\[ P = [P_1, \ldots, P_T]. \]  

where \( P_t, \forall t = 1 \ldots T \) shows the output power at time \( t \). It should be clarified that the vectors mentioned in (3) and (4) will be determined based on optimization functions based on IGDT.

2.4. Profit function

The profit function of a unit for a period of \( T \) hours is calculated by the following formula:

\[ F(P, \lambda) = P \cdot \lambda - \sum_{t=1}^{T} \{ C_i(P_t) + C_{\text{startup}} \cdot U_i \cdot (1 - U_{t-1}) + C_{\text{shutdown}} \cdot (1 - U_t) \cdot U_{t-1} \} \]

where superscript “" indicates the transpose function.

The profit is defined as the revenue from the sale of energy minus the production costs and start up/shutdown cost of the unit. The operation cost of the available generation thermal unit can be represented as follows:

\[ C_i(P_t) = a + bP_t + c(P_t)^2 \]

The operating constraints of the generation unit include the following case:

\[ \begin{align*}
\text{PC}_{\text{min}} \cdot U_t & \leq P_t \leq \text{PC}_{\text{max}} \cdot U_t; \forall t = 1, \ldots, T \\
P_t - P_{t-1} & \leq R^P \cdot U_t; \forall t = 1, \ldots, T \\
P_{t-1} - P_t & \leq R^\downarrow \cdot U_{t-1}; \forall t = 1, \ldots, T \\
|X^\text{on}_{t-1} - R^\uparrow| & \cdot |U_{t-1} - U_t| \geq 0; \forall t = 1, \ldots, T \\
|X^\text{off}_{t-1} - R^\downarrow| & \cdot |U_t - U_{t-1}| \geq 0; \forall t = 1, \ldots, T
\end{align*} \]

The minimum and maximum outputs of the unit are represented using (7). Eqs. (8) and (9) describe the ramping rate limits of generation unit. Finally, the minimum up/down time constraints are expressed as (10) and (11), respectively.
2.5. Uncertainty model

For a price-taker generation company, the market price is an uncertain variable. Thus, the uncertainty model of market price is necessary and needs to be considered since it has direct impact on the expected profit and optimal bidding strategy. In this part, a model is suggested for representing the uncertainty of market prices. The related robustness and opportunity functions will be derived based on this model.

Here, the market price of electricity is uncertain for which the following Weighted Average Squared Error (WASE) information gap model is considered:

\[ U(x, \lambda) = \{ \lambda : eW^{-1}e \leq \lambda^2 \}; \lambda \geq 0 \]  \tag{12}

where \( x \) and \( W \) represent uncertainty variable and variance–covariance matrix between hourly market prices, respectively. Note that the variance–covariance matrix \( W \) can be estimated using historical data. This model states that the WASE of price forecasting is not greater than \( \lambda^2 \). It should be emphasized that WASE allows for the convenient expression of the deviations of actual price from the expected values, which in turn allows for making robust and/ or beneficial decisions concerning these deviations.

2.6. Deriving robustness function

The function \( \tilde{z}(P, F_k) \) is related to the profit lower than the maximum profit and works as a risk-averse mechanism. In other words, \( \tilde{z}(P, F_k) \) measures the protection level of the decision against experiencing low profits. Thus, a low value of this function, \( F_k \), is expected that \( \tilde{z}(P, F_k) \) increases with the decrease of \( F_k \). Note that this condition represents the worst profit for the producer.

As mentioned, the robustness function \( \tilde{z}(P, F_k) \) is related to low market prices and represents the largest value of the uncertainty variable in a way that the minimum value of the electricity selling profit is greater than a desired profit target \( F_k \). \( \tilde{z}(P, F_k) \), could be computed according to the procedure described as follows.

Inserting (2) in (5) to find the minimum value of the profit yields

\[ \min_{P, e} F(P, e) = P \cdot (\tilde{\lambda})^T + \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} - F_k \]  \tag{13}

Subject to: \( eW^{-1}e \leq \lambda^2 \) \tag{14}

where \( \lambda^2 \) is a variable bounding the value of the WASE. Eqs. (13) and (14) can be rewritten as follows:

\[ \min_{P, e} F(P, e) = P \cdot (\tilde{\lambda})^T + \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} + C_{\text{shutdown}} \cdot (1 - U_t) \cdot U_{t-1} \] \tag{15}

Since the optimization problem is convex, the first-order optimality condition for the associated Lagrangian problem (15) can be stated as

\[ \nabla_{\epsilon e} \{ P \cdot e + \mu (\lambda^2 - eW^{-1}e) \} = 0 \]  \tag{16}

where \( \mu \) is the Lagrangian multiplier of the constraint (14). Performing the derivations of (16) results in

\[ \epsilon > 1 \lambda^2 = \frac{1}{\mu} WP \] \tag{17}

Combination with (17) and assuming \( W = WP \) yields

\[ \lambda^2 = \frac{1}{4\mu} WP \]  \tag{18}

And

\[ \frac{1}{2\mu} = \frac{\pm\lambda}{\sqrt{WP}} \]  \tag{19}

Substituting (19) in (17) results in:

\[ \epsilon = \frac{\pm\lambda}{\sqrt{WP}} \]  \tag{20}

Thus, the lowest profit for worst-case error the “−” sign is selected in (20) is

\[ \min F(P, \epsilon) = P \cdot (\tilde{\lambda})^T - \lambda \sqrt{WP} - \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} + \lambda \sqrt{WP} \]  \tag{21}

Using (21), \( \epsilon \) can be computed for a given profit target \( F_k \) as

\[ \epsilon(P, F_k) = \frac{P \cdot (\tilde{\lambda})^T - \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} - F_k}{\sqrt{WP}} \]  \tag{22}

Note that \( F_k \) represents the profit target which the producer is willing to face. Then, the robustness function and, therefore, a robust bidding strategy (risk-averse model) can be derived by providing the largest possible \( \epsilon \) (worst case) for a given profit target \( F_k \), as shown in (23).

\[ \hat{z}(P, F_k) = \max_{P, \epsilon} \{ P \cdot (\tilde{\lambda})^T - \lambda \sqrt{WP} - \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} - F_k \} \]  \tag{23}

Expression (23) represents the robustness function which should be maximized subject to the constraints (6)–(11).

2.7. Deriving opportunity function

The opportunity function \( \hat{b}(P, F_w) \) is related to high market prices or high electricity selling profit and evaluates the possibility of high profit from high prices. In other words, this is the immunity bidding strategy (risk-averse model) can be derived by providing the largest possible \( x \) (best case) for a given profit target \( F_w \), as shown in (24).

\[ \hat{b}(P, F_w) = \min_{P, \epsilon} \{ P \cdot (\tilde{\lambda})^T + \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} + \lambda \sqrt{WP} \} \]  \tag{24}

where \( F_w \) is generally bigger than \( F_k \). The highest profit can be defined using the “+” sign in (21) for the best-case error as

\[ \max F(P, \epsilon) = P \cdot (\tilde{\lambda})^T + \sum_{t=1}^{T} \{ C_t(P_t) + C_{\text{startup}} \cdot (1 - U_t) \cdot U_{t-1} \} + \lambda \sqrt{WP} \]  \tag{25}

Similar to the robustness function using (25) for a given profit target \( F_w \), the \( \epsilon \) can be computed. Note that \( F_w \) represents the highest electricity selling profit with which the producer is willingly to be faced. Then, using (25), the opportunity function can be derived for a given profit target \( F_w \), as shown in (26).
\[ \beta(p, F_w) = \min_{\gamma} (p, F_w) \]
\[ F_w - P(\gamma) - \sum_{i=1}^{T} \left( C_i(p) + C_{ramp} U_j \left( 1 - U_{j-1} \right) + C_{max} \left( 1 - U_j \right) U_{j-1} \right) \]
\[ = \min_{\gamma} \frac{1}{\sqrt{P_{\text{max}}}} \]
\[ (26) \]

The constraints are the same as those of the robustness function which are represented in (6)–(11). It is expected that \( \beta(p, F_w) \) increases with the increase of \( F_w \).

The robustness and opportunity functions are mixed-integer nonlinear programming problems which can be solved using SBB/CONOPT [36] under GAMS [37]. Table 1 provides the size of each problem, which is expressed as the number of binary variables, real variables and constraints.

3. Solution methodology

The optimal bidding strategy problem is formulated as a mixed-integer nonlinear problem and can be solved using SBB/CONOPT [36] under GAMS [37]. The procedure of simulation and result derivation is as follows.

1. The profit maximization problem is simulated based on (5) and constraints (6)–(11) by considering the forecasted market price data.
2. For some values of \( F_k \) with a fixed step, which is less than the maximum profit, the robustness function (23) with constraints (6)–(11) is simulated and the results represent the robustness level.
3. For some values of \( F_w \) with a fixed step, which is higher than the maximum profit, the opportunity function (26) with constraints (6)–(11) is simulated and the results are derived.
4. While for some values of \( F_k \) and \( F_w \), values of \( P, U \) and \( \alpha(p, F_k) \) and \( \beta(p, F_w) \) are derived, the day-ahead prices are calculated using (20) and (2) by instituting the \( \alpha(p, F_k) \) and \( \beta(p, F_w) \) instead of \( \alpha \), respectively. Then, the output power and day-ahead price pairs are used to build the step-wise bidding strategy.

4. Case study

In this section, numerical simulations are conducted to illustrate the work of the proposed method. The thermal unit producer participated in a day-ahead market; thus, the time horizon of this study contains 24 periods (1 day).

4.1 Data

The data of the generation thermal unit and technical constraint are provided in Table 2. The variance–covariance matrix for the day-ahead market is provided in Table 3. The prices of working days in the Iberian market from January 2008 to October 2008 are used to construct the variance–covariance matrices using appropriate MATLAB functions. The price profile of one day of Iberian market is considered as the forecasted market price profile for the study horizon, which is depicted in Fig. 1. In these simulations, the profit step for \( F_k \) and \( F_w \) is considered equal to $600.

The problem associated with a single instance of profit target, as characterized by 24 binary variables, 24 real variables and 144 constraints, is solved using an Intel (R) Celeron (R) CPU, 2-GHz and 0.99 GB of RAM computer in the average solution time of about 20 ms.

4.2 Results

The deterministic expected value problem consisting of maximizing the profit (5) subject to the constraints (6)–(11) and considering the market price forecasts is provided in Fig. 1. The maximum profit in this case is equal to $9582.50. Note that the value of the robustness and opportunity functions related to the maximum profit is equal to zero, i.e. \( \gamma(9582.50) = \beta(9582.50) = 0 \). Also, the hourly expected profit considering deterministic price profile is shown in Fig. 2. In some hours, the generating unit is going to shut-down mode and so, the related profit is negative.

The results of simulations related to the robustness function are useful if the producer decided on a risk-averse strategy and the opportunity function-related strategy can be used when the producer chose to be in a risk-taking decision making mode.

4.3. Results of robustness function

The robustness \( \gamma(P, F_k) \) versus electricity selling profit targets \( F_k \) are depicted in Fig. 3, in which it is clear that robustness increases with the decrease \( F_k \) as expected. If the generation company desires risk-averse decision and higher robustness bidding strategy, less profit should be obtained and vice versa; if the generation company obtains less profit, its decision will be more robust.

For example, if the profit target \( F_k \) is equal to $8382.5, according to Fig. 3, it is clear that the \( \gamma(P, 8382.5) = 0.273 \), which means that if the weighted square error of the forecasted hourly market prices equals to 0.273, therefore, the profit of the generation company with this bidding strategy will not be less than $8382.5.

In the risk-averse decision, the day-ahead price errors are calculated using \( \gamma(P, F_k) \) values by instituting the \( \gamma(P, F_k) \) instead of \( \alpha \) in Eq. (20). After computing of errors, the day-ahead prices in the risk-averse decision are calculated using calculated errors and Eq. (2).

Using the calculated prices and the output power in the risk-averse decision, Fig. 4 shows \( F(P(\gamma(F_k))) \) and \( F(P(\gamma(F_k)) = 0) \), \( \gamma(\gamma(F_k))) \) curves versus profit target \( F_k \). This figure illustrates the robustness expected profit and the expected profit versus electricity selling profit targets \( F_k \). It is obvious that the robustness expected profit is higher than the expected profit. The difference between \( F(P(\gamma(F_k))), \gamma(\gamma(F_k))) \) and \( F(P(\gamma(F_k)) = 0), \gamma(\gamma(F_k))) \) shows the extra profit of using robust bidding curve instead of the bidding without considering the price uncertainty.
Table 3
Variance–covariance matrix for the day-ahead market.

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Fig. 1. Forecasted price data for the study horizon.

Fig. 2. Hourly expected profit considering deterministic price profile.

Fig. 3. Robustness curve.

Fig. 4. The robustness expected profit and the expected profit versus profit targets (FK).
for \( \hat{x}(P, F_k) = 0.896 \), the profit target \( F_k \) equal to \$5982.50 and according to Fig. 4, it is clear that the robustness \( F(P(\hat{x}(F_k))) \), \( \hat{x}(\hat{x}(F_k))) \) is equal to \$4693.842 and the \( F(P(\hat{x}(F_k) = 0), \hat{x}(\hat{x}(F_k))) \) is equal to \$4114.085. Thus, the extra profit of the robustness strategy is equal to \$4693.842 - \$4114.085 = \$579.757.

4.4. Results of opportunity function

The opportunity value \( \hat{\beta}(P, F_w) \) versus the electricity selling profit targets \( F_w \) is shown in Fig. 5. This figure is relevant if the generation company decides to take a risk. According to Fig. 5, it is clear that the opportunity value increases as \( F_w \) increases and vice versa; if the generation company obtains high profit, its decision will be more risk-taker. For example, if the profit target \( F_w \) is equal to \$13182.5, according to Fig. 5, it is clear that the \( \hat{\beta}(P, F_w) = 0.739 \), which means that if the weighted square error of the forecasted hourly market prices equal to 0.739, therefore, the profit of selling energy will be at least equal to \$13182.5.

In the risk-taking decision, the day-ahead price errors are calculated using \( \hat{\beta}(P, F_w) \) values by substituting the \( \hat{\beta}(P, F_w) \) instead of \( \hat{x} \) in Eq. (20). After computing of errors, the day-ahead prices in the risk-
taking decision are calculated using calculated errors and Eq. (2). Using the calculated prices and output power in the risk-taking decision, Fig. 6 shows $F(P(\hat{\beta}(F_W)), \lambda(\hat{\beta}(F_W)))$ and $F(P(\hat{\beta}(F_W) = 0), \lambda(\hat{\beta}(F_W)))$ curves versus the profit target $F_W$, which illustrates the opportunity expected profit and the expected profit versus electricity selling profit targets $F_W$, respectively. It is obvious that the opportunity expected profit is higher than the expected profit. In other words, this figure provides the extra profit of the risk-taking, which is equal to the difference between $F(P(\hat{\beta}(F_W)), \lambda(\hat{\beta}(F_W)))$ and $F(P(\hat{\beta}(F_W) = 0), \lambda(\hat{\beta}(F_W)))$. For example, according to Fig. 5, for $\hat{\beta}(P, F_W) = 1.077$, the profit target $F_W$ is equal to $14982.50$ and according to Fig. 6, it is clear that the opportunity expected profit is equal to $16543.708$ and the expected profit is equal to $15814.069$. Thus, the profit of the risk is equal to $16543.708 – 15814.069 = 729.639$, which can be delivered, if the generation company use this bidding strategy.

4.5. Risk-based optimal bidding strategy curves

The resulting risk-based optimal bidding strategy curves for hours 7, 11, 16 and 21 are reported in Figs. 7–10, respectively. The $x$-axis represents the bidding price whereas the $y$-axis represents the amount of the power that the generation company bids to the day-ahead market. It can be observed that these curves are made using the results of robustness and opportunity functions, as indicated in these figures. The results of simulations related to the robustness function are useful if the producer decided on a risk-averse strategy and the opportunity function-related strategy can be used when the producer choose to be in a risk-taking decision making mode. The breaking line in Figs. 7–10 show that the right hand of this line is achieved from the opportunity and the left hand is achieved from the robustness function. The final number of blocks in the bidding strategy curves is an output of the proposed approach. Note that this number is always less than or equal to the number of different profit targets used for solving the problem. In this case study, 20 different profit targets are imposed. Finally, Figs. 11 and 12 show the calculated day-ahead prices and the output power of generation unit for three profit targets using expressions (20) and (2) and the value $\alpha$ obtained from solving (23) and (26) subject to (6)–(11), respectively.
5. Conclusion

This paper a price-taker generation company considered to participate in day-ahead electricity energy market. While making optimal bidding strategy for producer, factors such as characteristics of generator and market price uncertainty need to be considered because of having direct impact on the expected profit and bidding curve. The market price considered an uncertain variable and it is assumed that the generation company could forecast the market prices. In this study, the uncertainty model of market price is considered based on the concept of weighted average squared error using a variance–covariance matrix. Information gap decision theory is used to develop the optimal bidding strategy of generation companies. This paper demonstrated that risk-averse or risk-taking decisions might affect expected profit and bidding curve to day-ahead electricity energy market. A case study is used to illustrate the proposed approach.

Appendix A. IGDT background

Uncertainties may be harmful and lead to lower profits or may be beneficial and lead to higher profits. IGDT addresses these two conflicting issues using two immunity functions: robustness and opportunity.

An IGDT decision problem is specified by three components: objective function, performance requirements and uncertainty model. These issues are explained as follows.

A.1. Objective function

For a set of decision variables $P$, uncertainty variable $x$ and uncertain variable $\lambda$, the objective function $F(P, \lambda)$ expresses the input/output structure of the system for which the decision is applied. In this paper, the objective function is the electricity selling profit function with which a generation company is faced.

A.2. Performance requirements

The performance requirements which describe the requirements from the objective function can be expressed in terms of profit function. These requirements are evaluated based on robustness and opportunity functions. The functions for the electricity selling profit problem can be stated as follows:

$$\hat{x} = \max \{x : \text{minimum profit}\text{,(least profit) which is not less than a given profit target}\} \ \{\text{Robustness}\} \quad (A.1)$$

$$\hat{y} = \min \{x : \text{maximum profit}\text{,(best profit) which is bigger than a given profit target}\} \ \{\text{Opportunity}\} \quad (A.2)$$

The robustness function addresses the harmful face of the uncertainty and expresses the greatest level of uncertainty at which the electricity selling profit cannot be less than a given value $F_k$. In other words, this function describes the risk-aversion possibility of an optimal bidding strategy. This means that a large value of $\hat{x}$ is desirable. Therefore, it can be defined mathematically through an optimization problem as follows:

$$\hat{x}(P, F_k) = \max \{x : \min F(P, \lambda) \geq F_k\} \quad (A.3)$$

For a large value of $\hat{x}(P, F_k)$, the decision will be robust, risk-averse and insensitive to the uncertainties. On the other hand, if $\hat{x}(P, F_k)$ is small, the decision will be fragile and sensitive to the variation of uncertain variable; therefore, its related decision will not be more consistent.

The opportunity function addresses the propitious aspect of the uncertainty and evaluates the possibility of achieving profits. Here, $\hat{y}$ is the minimum value of $x$ which can be tolerated in order to enable the possibility of high electricity selling profit as a result of decisions, $P$. In other words, the opportunity function is the least value of $x$ for which the electricity selling profit can possibly be as high as a given value $F_w$. This function is the immunity against windfall profit. Thus, a high value of $\hat{y}(P, F_w)$ indicates a situation in which the profit is achievable in the presence of high market prices. This function is related to determining risk-taking optimal bidding strategy. The corresponding mathematical formulation can be represented through the following minimization problem:

$$\hat{y}(P, F_w) = \min \{x : \max F(P, \lambda) \geq F_r\}. \quad (A.4)$$

where $F_r$ is generally bigger than $F_k$.

A.3. Uncertainty model

The uncertainty is represented by an information gap model. The information gap model of uncertainty $U(x, \lambda)$ assumes the prior information about the uncertain vector $\lambda$. In this paper, it is assumed that the price of energy in the day-ahead market is uncertain. Also, the following weighted average squared error model is considered:

$$U(x, \lambda) = \{\lambda = \bar{\lambda} + e : eW^{-1}e \leq \bar{x}^2\}; \ x \geq 0. \quad (A.5)$$

References


